Price Calculations for a Reopened Treasury Inflation-Protected Security (TIPS)

The process involves several steps to obtain variables needed to populate the final formula. The steps are as follows:

1) Calculate CPI Reference Date Values (Ref CPI_{Date}, Index $Ratio_{Date}$).
2) Calculate the Present Value of the securities Cash Flows.
3) Calculate the amount of Accrued Interest.
4) Calculate the Nominal Price of the Security.
5) Calculate the Adjusted Price which adjusts the price to compensate for inflation.
6) Calculate the Net Price which includes Accrued Interest.

These examples are provided for illustrative purposes only and are in no way a prediction of interest rates or prices on any bills, notes or bonds issued by the Treasury.

In order for the reader to follow the step-by-step calculations, these examples were prepared on an Excel spreadsheet using 15 decimals, with rounding at each step. For readers who use multi-decimal calculators, we recommend setting the calculator to its maximum decimal settings and then applying normal rounding procedures.

In actual practice, Treasury uses a mainframe and generally does not round prior to determining the final result. In the case of any discrepancies due to rounding, determinations by the Treasury shall be final.

1) Calculations with regard to the Ref CPI_{Date} and the Index $Ratio_{Date}$ for a specific date will be rounded to five decimal places.
2) Accrued Interest is to be rounded to the 6th place.
3) Final Prices are rounded to the 6th place.

Variables / Inputs

Description: **US Treasury TIPS 3.625% 01/15/2008**

Issuance & Pay Date Information	
Dated Date:	01/15/1998
Issue Date: (Reopening)	10/15/1998
Maturity Date:	01/15/2008
Payment Dates:	1/15, 7/15
First Payment Date:	01/15/1999

Security Information		
C =	3.625	Real Coupon
i =	0.03650	Yield
n =	18	Periods
r =	92	(Oct. 15, 1998 to Jan. 15, 1999)
s =	184	(July 15, 1998 to Jan. 15, 1999)

CPI-U Data			
Ref $CPI_{DatedDate}$ =	161.55484	Jan. 15, 1998	
Ref CPI_{M} =	163.2	Jul. 98	
Ref CPI_{M+1} =	163.4	Aug. 98	
CPI_{Date} (Valuation Date):		Oct. 15, 1998	
D =	31	Total Days in	Oct. 98
t =	15	Valuation Day	Oct. 15, 1998

Solving for the Input Variables

The following variables need to be calculated in order to solve for price.
For the following formulas use Ref CPI_{Date} = Ref $CPI_{10/15/1998}$ and Index $Ratio_{Date}$ = Index $Ratio_{10/15/1998}$

Solving for Reference CPI_{Date}

Ref CPI_{Date} = Ref CPI_M + ((t - 1) / D) * (Ref CPI_{M+1} - Ref CPI_M)

Using variables from above, Ref $CPI_{10/15/1998}$ is calculated as:

1) Ref $CPI_{10/15/1998}$ = 163.2 + ((15 - 1) / 31) * (163.4 - 163.2)
2) Ref $CPI_{10/15/1998}$ = 163.2 + (0.451612903225806 * 0.2)
3) Ref $CPI_{10/15/1998}$ = 163.2 + (0.090322580645161)
4) Ref $CPI_{10/15/1998}$ = 163.29032 Rounded to 5 places

Solving for Reference Index $Ratio_{Date}$

Index $Ratio_{Date}$ = Ref CPI_{Date} / Ref $CPI_{DatedDate}$

Using variables from above, the Index $Ratio_{10/15/1998}$ is calculated as:

1) Index $Ratio_{10/15/1998}$ = 163.29032 / 161.55484
2) Index $Ratio_{10/15/1998}$ = 1.01074 Rounded to 5 places

Solving for Unadjusted Accrued Interest

A = ((s - r) / s) * (C / 2)

1) A = ((184 - 92) / 184) * (3.625 / 2)
2) A = (0.500000000000000 * 1.8125)
3) A = 0.906250 Rounded to 6 places

Solving for Adjusted Accrued Interest

A_{adj} = A * Index $Ratio_{Date}$

1) A_{adj} = 0.906250 * 1.01074
2) A_{adj} = 0.915983 Rounded to 6 places

Solve for Cash Flows Step 1

The following formula is used to calculate the Present Value of 1 due at the end of n periods. For n = 18.
v^n = 1 / (1 + i / 2) ^{n}

1) v^n = 1 / (1 + 0.03650 / 2)^{18}
2) v^n = 1 / (1 + 0.01825)^{18}
3) v^n = 1 / (1.01825)^{18}
4) v^n = 1 / 1.384776030642019
5) v^n = 0.722138438182219

Solve for Cash Flows Step 2

The following formula is used to calculate the Present Value of 1 period for n periods, in this case 18 periods.

$a_{\overline{n}|} = (1 - v^n) / (i / 2) = v + v^2 + v^3 + \ldots v^n$ (see *Special Case*)

Special Case: If i = 0, then $a_{\overline{n}|} = n$. Furthermore, when i = 0, $a_{\overline{n}|}$ cannot be calculated using the formula: $(1 - v^n) / (i/2)$. In the special case where i = 0, $a_{\overline{n}|}$ must be calculated as the summation of the individual present values (i.e., $v + v^2 + v^3 + \ldots + v^n$). Using the summation method will always confirm that $a_{\overline{n}|} = n$ when i= 0.

1) $a_{\overline{n}|}$ = (1 - 0.722138438182219) / (0.03650 / 2)

2) $a_{\overline{n}|}$ = 0.277861561817781 / 0.01825

3) $a_{\overline{n}|}$ = 15.225291058508548

Solving for Prices

After having calculated the necessary variables we can now solve for price by using the following formulas.

Solve for Nominal Price

Populate the formula with the values derived above. Then break the equation down into smaller parts as expressed by labels.

Equation

$$P = \frac{(C / 2) + (C / 2) * a_{\overline{n}|} + 100 * v^n}{1 + (r / s) * (i / 2)} - ((s - r) / s) * (C / 2)$$

Formula

P = (((C / 2) + ((C / 2) * $a_{\overline{n}|}$) + (100 * v^n)) / 1 + (r / s) * (i / 2)) - ((s - r) / s) * (C / 2)

P = (((3.625/2)+((3.625/2) * 15.225291058508548)+(100 * 0.722138438182219))/ 1 + (92/184)* (0.03650/2))-(((184-92)/ 184) * 3.625/2)

Part A | Part B | Part C | Part D | Part E

1) P = ((1.8125 + 27.595840043546743 + 72.213843818221900) / 1.009125000000000) - 0.906250

Part A + Part B + Part C = Part G = 101.622183861768643

Rounded to 6 places

Simplified Further

2) P = (Part G / Part D) - Part E

2) P = (101.622183861768643 / 1.009125000000000) - 0.906250

3) P = 100.703266554459203 - 0.906250

4) P = 99.797017 **Nominal Price** Rounded to 6 places

Solve for Adjusted Price

$P_{adj} = P \times \text{Index Ratio}_{Date}$

1) P_{adj} = 99.797017 x 1.01074

2) P_{adj} = 100.868837 **Adjusted Price** Rounded to 6 places

Solve for Settlement Amount

$SA = P_{adj} + A_{adj}$

1) SA = 100.868837 + 0.915983

2) SA = 101.784820 **Settlement Amount** Rounded to 6 places

Sample Settlement Information

If the 6-decimal Adjusted Price per hundred is 100.868837 and the 6-decimal Settlement Amount per hundred is 101.784820 and the 6-decimal Adjusted Accrued Interest per hundred is 0.915983, then:

Face Amount	1,000,000.00	100,000,000.00	1,000,000,000.00
Nominal Principal	997,970.17	99,797,017.00	997,970,170.00
Adjusted Price Amount	1,008,688.37	100,868,837.00	1,008,688,370.00
Adjusted Accrued Interest	9,159.83	915,983.00	9,159,830.00
Settlement Amount (Padj + Aadj)	1,017,848.20	101,784,820.00	1,017,848,200.00

Definitions Of Variables Used In This Example

P = unadjusted or real price per 100 (dollars)

P_{adj} = inflation adjusted price; P x Index $Ratio_{Date}$

A = unadjusted accrued interest per $100 original principal

A_{adj} = inflation adjusted accrued interest; A x Index $Ratio_{Date}$

SA = settlement amount including accrued interest in current dollars per $100 original principal; $P_{adj} + A_{adj}$

r = days from settlement date to next coupon date

s = days in current semiannual period

i = real yield, expressed in decimals (e.g., 0.0325)

C = real annual coupon, payable semiannually, in terms of real dollars paid on $100 initial, or real, principal of the security

n = number of full semiannual periods from issue date to maturity date, except that, if the issue date is a coupon frequency date, n will be one less than the number of full semiannual periods remaining until maturity. Coupon frequency dates are the two semiannual dates based on the maturity date of each note or bond issue. For example, a security maturing on July 15, 2026 would have coupon frequency dates of January 15 and July 15.

Date = valuation date

D = the number of days in the month in which Date falls

t = calendar day corresponding to Date

CPI = Consumer Price Index number

CPI_M = CPI reported for the calendar month M by the Bureau of Labor Statistics

Ref CPI_M = reference CPI for the first day of the calendar month in which Date falls, e.g., Ref CPI_{April1} is the $CPI_{January}$

Ref CPI_{M+1} = reference CPI for the first day of the calendar month immediately following Date